Progressions →

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$\log\frac{a}{b} = \log a - \log b$

$\log a^m = m\log a$

$\log\frac{x^n}{y^m} = n\log x - m\log y$

Q → Find sum of ...

$$\log a + \log\frac{a^2}{b} + \log\frac{a^3}{b^2} + \log\frac{a^4}{b^3} \cdots\cdots \text{n terms} \ldots$$

$$\neq \log\frac{a^n}{b^{n-1}}$$

$$\{\log a\} + \{2\log a - \log b\} + \{3\log a - 2\log b\} + \cdots\cdots \{n\log a - (n-1)\log b\}$$

$$\log a\{1+2+3+\cdots\cdots n\} - \log b\{1+2+\cdots\cdots+n-1\}$$

$$\log a\left\{\frac{n(n+1)}{2}\right\} - \log b\left\{\frac{n(n-1)}{2}\right\}$$

$$\frac{n}{2}\{(n+1)\log a - (n-1)\log b\}$$

$$\frac{n}{2}\{\log a^{n+1} - \log b^{n-1}\}$$

$$\frac{n}{2}\left\{\log\frac{a^{n+1}}{b^{n-1}}\right\}$$

Q2 → If $a, b, \& c$ are in AP $\Rightarrow$ $b-a = c-b = \boxed{d}$

Show

(i) $b+c,\ c+a,\ a+b$ are in AP

$a_2 - a_1 = (c+a) - (b+c)$

$= a-b = -d$

$a_3 - a_2 = (a+b) - (c+a)$

$= b-c = -d$

$\Rightarrow$ AP

a, b, c are in AP

$-(a+b+c)$ from all terms

$-(b+c),\ -(a+c),\ -(a+b)$ are in AP

$\times(-1)$ in all terms

$(b+c),\ (c+a),\ (a+b)$ are in AP

Hence proved...

(ii) ... are in AP

$b - a = c - b \neq d$

(ii) $\frac{1}{bc}, \frac{1}{ca}, \frac{1}{ab}$ are in AP

$a_2 - a_1 = \frac{1}{ca} - \frac{1}{bc} = \frac{b-a}{abc} = \frac{d}{abc} = D$

$a_3 - a_2 = \frac{1}{ab} - \frac{1}{ca} \Rightarrow \frac{c-b}{abc} \Rightarrow \frac{d}{abc} = D$

Hence proved..

$\boxed{b-a = c-b} = d$

a, b, c are in AP

$\times \frac{1}{abc}$ with all terms

$\frac{1}{bc}, \frac{1}{ac}, \frac{1}{ab}$ are in AP

(iii) $a^2(b+c), b^2(c+a); c^2(a+b)$ are in AP

Conditions $\boxed{b-a = c-b} = d$

① a, b, c are in AP

② $\sum ab = \boxed{ab+bc+ca \neq 0}$

$a_2 - a_1 = b^2(c+a) - a^2(b+c)$

$= b^2c + b^2a - a^2b - a^2c$

$= c(b^2 - a^2) + ab(b-a)$

$= (b-a)\{c(b+a) + ab\} = d\{ab+bc+ca\}$

$a_3 - a_2 = c^2(a+b) - b^2(c+a)$

$= c^2a + c^2b - b^2c - b^2a$

$= a(c^2 - b^2) + bc(c-b)$

$= (c-b)\{a(c+b) + bc\} = d\{ab+bc+ca\}$ Hence proved.

(iv) $a\left(\frac{1}{b}+\frac{1}{c}\right), b\left(\frac{1}{c}+\frac{1}{a}\right), c\left(\frac{1}{a}+\frac{1}{b}\right)$ are in AP.

Given a, b, c are in AP

$\boxed{b-a = c-b} = d$

$a_2 - a_1 = \left(\frac{b}{c}+\frac{b}{a}\right) - \left(\frac{a}{b} - \frac{a}{c}\right)$

$= \frac{b}{c} - \frac{a}{c} + \frac{b}{a} - \frac{a}{b}$

$$= \frac{b-a}{c} + \frac{b^2-a^2}{ab}$$

$$\Rightarrow (b-a)\left\{\frac{1}{c} + \frac{b+a}{ab}\right\} = d\left\{\frac{ab+bc+ac}{abc}\right\}$$

Similarly.... $a_3 - a_2 = (c-b)\left\{\frac{ab+bc+ac}{abc}\right\} = d\left\{\frac{ab+bc+ac}{abc}\right\}$

Hence proved...

(v) $\frac{1}{\sqrt{b}+\sqrt{c}}, \frac{1}{\sqrt{c}+\sqrt{a}}, \frac{1}{\sqrt{a}+\sqrt{b}}$ are in AP

$$\boxed{d = b-a = c-b}$$

$$a_2 - a_1 = \frac{1}{\sqrt{c}+\sqrt{a}} - \frac{1}{\sqrt{b}+\sqrt{c}} = \frac{(\sqrt{b}+\cancel{\sqrt{c}}) - (\cancel{\sqrt{c}}+\sqrt{a})}{(\sqrt{c}+\sqrt{a})(\sqrt{b}+\sqrt{c})}$$

$$= \frac{(\sqrt{b}-\sqrt{a})}{(\sqrt{c}+\sqrt{a})(\sqrt{b}+\sqrt{c})} \times \frac{\sqrt{b}+\sqrt{a}}{\sqrt{b}+\sqrt{a}}$$

$$= \frac{d}{(\sqrt{a}+\sqrt{b})(\sqrt{b}+\sqrt{c})(\sqrt{c}+\sqrt{a})}$$

Similarly.. $a_3 - a_2 = \frac{1}{\sqrt{a}+\sqrt{b}} - \frac{1}{\sqrt{c}+\sqrt{a}}$

$$= \frac{\sqrt{c}-\sqrt{b}}{(\sqrt{a}+\sqrt{b})(\sqrt{c}+\sqrt{a})} \Rightarrow \frac{d}{(\sqrt{a}+\sqrt{b})(\sqrt{b}+\sqrt{c})(\sqrt{c}+\sqrt{a})}$$

Hence proved...

Q → a^2, b^2, c^2 are in AP $\Rightarrow \boxed{b^2-a^2 = c^2-a^2} = d$

HW

① $\frac{1}{b+c}, \frac{1}{c+a}, \frac{1}{a+b} \Rightarrow$ AP

② $\frac{a}{b+c}, \frac{b}{c+a}, \frac{c}{a+b} \Rightarrow$ AP

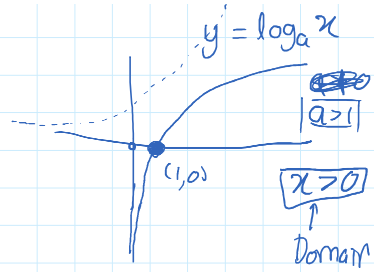
Q → If $\log_{10}2$, $\log_{10}(2^x-1)$ & $\log_{10}(2^x+3)$ are in AP

Options
$x=0$

then

$\times \quad \log_{10} 2, \ \log_{10}(0), \ \log_{10} 4$

$\to -\infty \qquad \times$



(1) $x=0$

(2) $x=1$

(3) $x=\log_2 5$

(4) $x=\log_{10} 2$

$$2b = a+c$$

$$2\log_{10}(2^x-1) = \log_{10}2 + \log_{10}(2^x+3)$$

$$\log_{10}(2^x-1)^2 = \log_{10}\left(2\times(2^x+3)\right)$$

Remove $\log_{10}$

$$(2^x-1)^2 = 2\cdot 2^x + 6$$

$$(2^x)^2 - 2(2^x)(1) + 1 = 2\cdot 2^x + 6$$

$$(2^x)^2 - 4(2^x) - 5 = 0$$

Lets say $2^x = z$

$$z^2 - 4z - 5 = 0$$

$$z^2 - 5z + z - 5 = 0$$

$$z(z-5) + 1(z-5) = 0$$

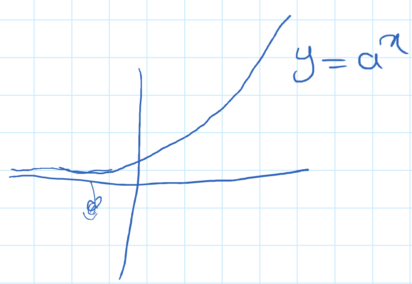
$$(z+1)(z-5) = 0$$

$$z=-1 \quad \text{or} \quad z=5$$

$$2^x = -1 \quad \text{or} \quad 2^x = 5$$

$\times \ \times \ \times$

$$\boxed{x = \log_2 5}$$



Q → $\boxed{x^{18} = y^{21} = z^{28}} = k$

$$\log_a b = \frac{1}{\log_b a}$$

$$\log_a b = \frac{\log_\theta b}{\log_\theta a}$$

Show $3, \ 3\log_y x, \ 3\log_z y, \ 7\log_x z$ are in AP

$$\log x^{18} = \log y^{21} = \log z^{28} = \log k$$

$$\log x^{18} = \log y^{21} = \log z^{28} = \log k$$

$$\boxed{18\log x = 21\log y = 28\log z = \log k} \quad (1)$$

$$a_2 - a_1 = 3\log_y x - 3$$

$$\Rightarrow 3\frac{\log x}{\log y} - 3 = 3\left(\frac{21}{18}\right) - 3 = \frac{7}{2} - 3 \Rightarrow \frac{1}{2}$$

$$a_3 - a_2 = 3\log_z y - 3\log_y x$$

$$= 3\left(\frac{\log y}{\log z} - \frac{\log x}{\log y}\right)$$

$$= 3\left(\frac{28}{21} - \frac{21}{18}\right) = 3\left(\frac{4}{3} - \frac{7}{6}\right)$$

$$\Rightarrow 4 - \frac{7}{2} = \frac{1}{2}$$

$$a_4 - a_3 = 7\log_x z - 3\log_z y$$

$$= 7\frac{\log z}{\log x} - 3\frac{\log y}{\log z}$$

$$= 7\left(\frac{18}{28}\right) - 3\left(\frac{28}{21}\right)$$

$$\Rightarrow \frac{9}{2} - 4 = \frac{1}{2}$$

Hence a_1, a_2, a_3, a_4 are in AP

Q. $S = 1 + (1+x) + (1+x+x^2) + \cdots\cdots + (1+x+x^2+x^3\cdots+x^{n-1})$

$$a_n = 1 + x + x^2 + \cdots + x^{n-1}$$

$a = 1$, $r = x$, n

$$a_n = 1\frac{(x^n - 1)}{(x-1)}$$

$$S_n = a\frac{(r^n - 1)}{(r-1)}$$

$$\boxed{a_n = \frac{x^n - 1}{x - 1}}$$

$$S = \sum_{r=1}^{n} a_r$$

$$\frac{x^4 - 1}{x - 1} = x^3 + x^2 + x + 1$$

$$S = \sum_{r=1}^{n} \frac{x^r - 1}{x - 1}$$

$$S = \frac{x-1}{x-1} + \frac{x^2-1}{x-1} + \frac{x^3-1}{x-1} + \cdots + \frac{x^n-1}{x-1}$$

$$S = \frac{x-1+x^2-1+x^3-1 \cdots + x^n-1}{x-1}$$

$$S = \frac{(x+x^2+\cdots+x^n) - (\underbrace{1+1\cdots 1}_{n \text{ terms}})}{x-1}$$

$a = x$
$r = x$
n

$$S = \frac{x\frac{(x^n-1)}{(x-1)} - n}{x-1}$$

$$\boxed{S = \frac{x(x^n-1)}{(x-1)^2} - \frac{n}{(x-1)}}$$

Q.

$$\sum_{r=1}^{20} i^r = \underbrace{i+i^2+i^3+i^4}_{} + \cdots\cdots + i^{20}$$

$= 0$

$a = i$
$r = i$
$n = 20$

$$= \frac{i(i^{20}-1)}{(i-1)} = \frac{i(1-1)}{(i-1)} = 0$$

i
$i^2 = -1$
$i^3 = -i$
$i^4 = 1$

Sum = 0
4 Consecutive terms

Q.

$$\sum_{n=1}^{13} (i^n + i^{n+1})$$

$$(i+i^2) + (i^2+i^3) + \cdots\cdots + (i^{13}+i^{14})$$

$$(\underbrace{i+i^2+i^3+\cdots+i^{13}}_{13 \text{ terms}}) + (\underbrace{i^2+i^3+\cdots+i^{14}}_{13 \text{ term}})$$

$$\boxed{i^4 = 1}$$

$$i\left(\frac{i^{13}-1}{i-1}\right) + i^2\left(\frac{i^{13}-1}{i-1}\right)$$

$i^4 = 1$

$i^{13} = i^{12} i = 1 \times i = i$

$i\left(\frac{i-1}{i-1}\right) + i^2\left(\frac{i-1}{i-1}\right)$

$\frac{i+i^2}{i-1}$

~~$S \Sigma\ 1\cdot2\cdot3 + 2\cdot3\cdot4 +$~~

Q.

$$S_n = \frac{1}{1\cdot2\cdot3} + \frac{1}{2\cdot3\cdot4} + \frac{1}{3\cdot4\cdot5} + \cdots\cdots + \overbrace{\frac{1}{n(n+1)(n+2)}}^{a_n}$$

Partial fraction

Q.

$$\frac{px^2+qx+r}{(x+a)(x+b)(x+c)} = \frac{A}{(x+a)} + \frac{B}{(x+b)} + \frac{C}{(x+c)}$$

$$a_n = \frac{1}{n(n+1)(n+2)} = \frac{A}{n} + \frac{B}{n+1} + \frac{C}{n+2}$$

$$\frac{1}{\cancel{n(n+1)(n+2)}} \Rightarrow \frac{A(n+1)(n+2) + Bn(n+2) + C(n)(n+1)}{\cancel{n(n+1)(n+2)}}$$

$$1 = A(n^2+3n+2) + B(n^2+2n) + C(n^2+n)$$

$$0n^2 + 0n + 1 = n^2(A+B+C) + n(3A+2B+C) + 2A$$

Constant term

$2A = 1 \Rightarrow \boxed{A = \frac{1}{2}}$ — (a)

Linear terms

$0 = 3A+2B+C \Rightarrow \boxed{2B+C = -\frac{3}{2}}$

Quadratic term

$\boxed{B = -1}$ — (b)

$C = \frac{1}{2}$ — (c)

$0 = A+B+C \Rightarrow B+C = -1$

Partial fractions...

$$\frac{px+q}{(x+a)(x+b)} = \frac{A}{x+a} + \frac{B}{x+b}$$

$$\frac{1}{x^2-1}$$

$$\frac{1}{(x-1)(x+1)} = \frac{A}{(x+1)} + \frac{B}{(x-1)}$$

$$\frac{1}{\cancel{(x-1)(x+1)}} = \frac{A(x-1)+B(x+1)}{\cancel{(x+1)(x-1)}}$$

$$1 = x(A+B) - A + B$$

$A+B = 0$

$-A+B = 1$

$B = \frac{1}{2}$

$A = -\frac{1}{2}$

$$\frac{1}{x^2-1} = \frac{-\frac{1}{2}}{x+1} + \frac{\frac{1}{2}}{x-1} \quad ??$$

$$\Rightarrow \frac{1}{2}\left\{-\frac{1}{x+1} + \frac{1}{x-1}\right\}$$

$$C = \frac{1}{2}$$

$$0 = A + B + C \Rightarrow \boxed{B + C = -\frac{1}{2}}$$

$$\Rightarrow \frac{1}{2}\left\{ -\frac{1}{x+1} + \frac{1}{x-1} \right\}$$

$$\Rightarrow \frac{1}{2}\left\{ \frac{-(\cancel{x}+1) + (\cancel{x}+1)}{x^2-1} \right\}$$

$$\Rightarrow \frac{1}{\cancel{2}}\left(\frac{2}{x^2-1}\right) = \frac{1}{x^2-1}$$

$$\frac{1}{n(n+1)(n+2)} = \frac{\frac{1}{2}}{n} + \frac{-1}{n+1} + \frac{\frac{1}{2}}{n+2}$$

$$a_n \Rightarrow \frac{1}{2}\left\{ \frac{1}{n} - \frac{2}{n+1} + \frac{1}{n+2} \right\}$$

$$\boxed{a_n \Rightarrow \frac{1}{2}\left(\left\{\frac{1}{n} - \frac{1}{n+1}\right\} + \left\{\frac{1}{n+2} - \frac{1}{n+1}\right\} \right)}$$

$$a_1 + a_2 + \cdots + a_n$$

$$\cancel{\frac{1}{2}\left(\frac{1}{1} - \frac{2}{2} + \frac{1}{3}\right) + \frac{1}{2}\left(\frac{1}{2} - \frac{2}{3} + \frac{1}{4}\right)} \cdots$$

$$S_n = \frac{1}{2}\left\{ \left(\frac{1}{1} - \frac{1}{2}\right) + \left(\frac{1}{3} - \frac{1}{2}\right) \right\} + \frac{1}{2}\left\{ \left(\frac{1}{2} - \frac{1}{3}\right) + \left(\frac{1}{4} - \frac{1}{3}\right) \right\}$$

$$+ \frac{1}{2}\left\{ \left(\frac{1}{3} - \frac{1}{4}\right) + \left(\frac{1}{5} - \frac{1}{4}\right) \right\} \cdots\cdots + \frac{1}{2}\left\{ \left(\frac{1}{n} - \frac{1}{n+1}\right) + \left(\frac{1}{n+2} - \frac{1}{n+1}\right) \right\}$$

$$S_n = \frac{1}{2}\left\{ 1 - \frac{1}{2} + \cancel{\frac{1}{3}} - \cancel{\frac{1}{2}} + \cancel{\frac{1}{2}} - \cancel{\frac{1}{3}} + \cancel{\frac{1}{4}} - \cancel{\frac{1}{3}} + \cancel{\frac{1}{3}} - \cancel{\frac{1}{4}} + \cancel{\frac{1}{5}} - \cancel{\frac{1}{4}} + \cdots\cdots \right.$$

$$\left. \cdots \cancel{\frac{1}{n-1}} - \cancel{\frac{1}{n}} + \cancel{\frac{1}{n+1}} - \cancel{\frac{1}{n}} + \cancel{\frac{1}{n}} - \cancel{\frac{1}{n+1}} + \frac{1}{n+2} - \frac{1}{n+1} \right\}$$

$$S_n = \frac{1}{2}\left\{ \frac{1}{2} + \frac{(n+1) - (n+2)}{(n+2)(n+1)} \right\}$$

$$\boxed{S_n = \frac{1}{2}\left\{ \frac{1}{2} - \frac{1}{(n+1)(n+2)} \right\}}$$

$n \to \infty$ Second term $\to 0$

$$S_\infty = \frac{1}{4}$$

$$S_n = \frac{1}{2}\left\{ \frac{(n+1)(n+2) - 2}{2(n+1)(n+2)} \right\} \Rightarrow \frac{n^2 + 3n + \cancel{2} - \cancel{2}}{4(n+1)(n+2)} \to \frac{n(n+3)}{4(n+1)(n+2)}$$

Q. HW Find sum of n terms

$$\frac{1}{1\cdot3\cdot5} + \frac{1}{3\cdot5\cdot7} + \frac{1}{5\cdot7\cdot9} + \cdots \underset{a_n}{\frac{1}{(2n-1)(2n+1)(2n+3)}}$$

1, 3, 5 - - - $\quad 1+(n-1)2 = 2n-1$

3, 5, 7 - - - $\quad 3+(n-1)2 = 2n+1$

5, 7, 9 - - - $\quad 5+(n-1)2 = 2n+3$

$$\frac{1}{(2n-1)(2n+1)(2n+3)} = \frac{A}{2n-1} + \frac{B}{2n+1} + \frac{C}{(2n+3)}$$